Time-Varying Fine-Structure Constant Requires Cosmological Constant

Rainer W. Kühne

Vorm Holz 4, 42119 Wuppertal, Germany

Webb et al. presented preliminary evidence for a time-varying fine-structure constant. We show Teller's formula for this variation to be ruled out within the Einstein-de Sitter universe, however, it is compatible with cosmologies which require a large cosmological constant.

PACS numbers: 95.30.Sf, 95.30.Dr, 98.80.Es

The possibility of time-varying physical constants was suggested by Dirac[1,2] and Milne[3]. This suggestion has been widely discussed[4–10] for a number of motivations:

(i) The assumption of physical quantities being constant in space and time is ad hoc. It has to be either confirmed or rejected by observation[1,2].

(ii) The possibility of life as we know it depends on the values of a few basic physical constants and appears to be sensitive to their numerical values. This argument appeals to the anthropic principle[6,8,9].

(iii) The numerical values of the basic dimensionless constants, e. g. the fine-structure constant, are not yet explained. If they depend on other constants and the age of the universe and are therefore time-varying, then the actual number of the independent free parameters of the standard theory of particle physics would be reduced.

Products and quotients of physical constants have been shown to be of fundamental importance for new physical phenomena:

(i) The unit of flux quantization in superconductors is $\phi = h/(2e)$, where h denotes the Planck constant and e the unit electric charge[11,12]. This finding was the first experimental proof of the famous BCS-theory[13]. Furthermore, the flux quantization in units of ϕ is of central importance for the Josephson junctions[14].

(ii) The characteristic magnetic flux in the Aharonov-Bohm effect[15,16] is quantized in units of $\phi = h/e$.

(iii) The unit conductance in the integer quantum Hall effect[17] is $\hat{\sigma} = e^2/h$. In the fractional quantum Hall effect[18] the conductance is $\sigma = pe^2/h$, where p is a rational number.

(iv) The Chandrasekhar limit mass[19] of white dwarf stars is

$$M_c = \frac{3\sqrt{\pi}}{2\mu^2 M_p^2}\left(\frac{\hbar c}{G}\right)^{3/2}, \tag{1}$$

where $\hbar = h/(2\pi)$, c is the speed of light, G is Newton's gravitational constant, M_p is the proton rest mass, and μ is the number ratio of nucleons and electrons.

In order to find further fundamental and important physical phenomena one is attempted to try to multiply or divide the Planck units[20],

$$l_p = (G\hbar/c^3)^{1/2} \tag{2}$$
$$t_p = (G\hbar/c^5)^{1/2} \tag{3}$$
$$m_p = (\hbar c/G)^{1/2}, \tag{4}$$

with the Einstein constant[21],

$$\kappa = 8\pi G/c^4, \tag{5}$$

the "natural units" $\hbar$ and c, and the present value H_0 of the Hubble parameter[22]. By just doing this, Teller[5,10] found the remarkable relation (in modern notation),

$$\kappa m_p H_0 c = \kappa\hbar H_0/l_p = 8\pi t_p H_0 = \exp(-1/\alpha_0), \tag{6}$$

where

$$\alpha_0 = \frac{e^2}{4\pi\varepsilon_0\hbar c} \tag{7}$$

is the fine-structure constant. Here and in the following the subscript "0" refers to the present value of the respective parameter.

From Teller's equation one can derive the change rate

$$\frac{\dot{\alpha}_z}{\alpha_z} = \alpha_z\frac{\dot{H}_z}{H_z}, \tag{8}$$

where the subscript "z" denotes the value of the respective parameter at cosmological redshift z and the dot denotes the derivative with time. This rate can be determined experimentally. By examining quasar absorption lines in the redshift region $0.5 < z < 1.6$, Webb et al.[23] found preliminary evidence for a time-varying fine-structure constant,

$$\frac{\Delta\alpha}{\alpha} = (-1.09 \pm 0.36)\times 10^{-5}, \tag{9}$$

implying

$$\frac{\dot{\alpha}_z}{\alpha_z} = -1\times 10^{-5}H_z. \tag{10}$$

If the preliminary finding by Webb et al. will not be confirmed by future investigations, then Eq. 10 should be considered as an upper limit for the change rate of the fine-structure constant.

It is our aim to examine which cosmologies are compatible with both Teller's equation and (the upper limit of) the change rate of the fine-structure constant determined by Webb et al.

Compatibility is satisfied for cosmologies like the de Sitter universe[24] and the steady state cosmology[25,26],

where the Hubble parameter is constant in time. Unfortunately, these models have already been ruled out as viable cosmologies, because they contradict a number of other cosmological findings[27].

According to general agreement the expansion of the universe is explained by the Friedmann-Lemaître equation[28–30],

$$H^2 = \frac{8\pi G\varrho}{3} - \frac{kc^2}{R^2} + \frac{\lambda c^2}{3}. \tag{11}$$

The curvature radius R and the mean mass density ϱ scale with the redshift as,

$$R = R_0/(1+z) \tag{12}$$
$$\varrho = \varrho_0(1+z)^3. \tag{13}$$

By defining the present values of the mass parameter

$$\Omega_0 = \frac{8\pi G\varrho_0}{3H_0^2} \tag{14}$$

and the cosmological parameter

$$\lambda_0 = \frac{\lambda c^2}{3H_0^2}, \tag{15}$$

the Friedmann-Lemaître equation can be written as[31],

$$\left(\frac{H_z}{H_0}\right)^2 = \Omega_0(1+z)^3 - (\Omega_0 + \lambda_0 - 1)(1+z)^2 + \lambda_0. \tag{16}$$

The Einstein-de Sitter universe[32], where $\Omega_0 = 1$ and $\lambda_0 = 0$, is a special case of the Friedmann-Lemaître universe. In this cosmology, Eq. 16 is reduced to

$$\left(\frac{H_z}{H_0}\right)^2 = (1+z)^3. \tag{17}$$

By using this equation Dyson[7,10] derived from Teller's equation the formula

$$\alpha_z = \frac{\alpha_0}{1 - \frac{3}{2}\alpha_0\ln(1+z)}, \tag{18}$$

which somewhat resembles expressions known from running coupling constants in quantum field theory. Dyson's equation (18) yields the rate

$$\frac{\dot{\alpha}_z}{\alpha_z} = -\frac{3}{2}\alpha_z H_z \tag{19}$$

and is, unfortunately, ruled out by the experimental result of Webb et al.[23]

By regarding Eq. 16 we find Teller's equation to be compatible with the observation by Webb et al. if the present value Ω_0 of the mass parameter is extremely small and the cosmological parameter λ_0 slightly above unity.

The lower bound on Ω_0 is given by the baryonic matter content of the universe. The visible baryonic matter of the universe was determined to be[33]

$$\Omega_0 = 0.003h^{-1}, \tag{20}$$

where $h = H_0/(100\text{km}/(\text{s Mpc}))$. (The parameter h was introduced into cosmology for ease of notation; it should not be confused with the Planck constant.) This extremely small value of Ω_0 is compatible with other determinations of Ω_0 derived from examinations of the dynamics of clusters of galaxies if majority of their mass consists of a network of cosmic strings. This hypothetical network contributes to the local mass but do not influence the expansion of the universe[34]. Consequently, the mass contribution of the cosmic string network does not appear in the Friedmann-Lemaître equation. Furthermore, determinations of Ω_0 from hot big bang nucleosynthesis are model-dependent (especially the degree of inhomogeneities in the early universe is not well understood), for a discussion see Refs. 35 and 36.

If the observation by Webb et al. will be confirmed and Teller's equation is correct, then we can predict the following present values of cosmological parameters:

$$H_0 = 69.7 \text{ km}/(\text{s Mpc}) \tag{21}$$
$$\Omega_0 = 0.004 \tag{22}$$
$$\lambda_0 = 1.004. \tag{23}$$

This value of H_0 is compatible with majority of its recent determinations, e. g. Pierce et al.[37] found $H_0 = (87 \pm 7) \text{ km}/(\text{s Mpc})$, Freedman et al.[38] reported $H_0 = (80 \pm 17) \text{ km}/(\text{s Mpc})$, and Tanvir et al.[39] measured $H_0 = (69 \pm 8) \text{ km}/(\text{s Mpc})$. Harris et al.[40] found $H_0 = (77 \pm 8) \text{ km}/(\text{s Mpc})$ and Madore et al.[41] reported $H_0 = (73 \pm 21) \text{ km}/(\text{s Mpc})$.

Interestingly, the cosmological parameters of the Bonn-Potsdam model[31,42,43] resemble those suggested above. Recently, van de Bruck[44,45] has examined the effects of the hypothetical cosmic string network on the cosmological velocity field within the framework of this model and has shown them to be compatible with cosmological observations.

The possible time-variation of the fine-structure constant implies the variation of at least one of the constants c, $\hbar$, ε_0, and e. A time-varying speed of light, for example, would imply a change in the radiation losses of accelerated charges, which would be of importance especially in the very early universe[46].

In practice, time-variations of quantities with nonzero dimension can be measured and defined only relative to other quantities which have the same dimension. We will suggest a convention where the most fundamental constants are defined as being constant in time and can serve as a reference for the possible time-variation of other quantities.

We suggest to consider the "natural units" c, $\hbar$, k_B, and ε_0 as fundamental and constant in time. This is

because according to relativity, lengths are measured as time intervals multiplied with the speed of light. Even our unit length, the meter, is defined as the length which the speed of light covers in 1/299 792 458 seconds. Furthermore, quantum phenomena are described either by the particle picture or the wave picture, respectively. The "conversion factor" of some of the quantities of this duality is $\hbar$. Finally, temperature is a macroscopic quantity and can be defined by the energy of microscopic objects divided by the Boltzmann constant k_B.

According to concepts of quantum gravity, the Planck units, especially the Planck time, are most fundamental. Hence, we suggest to measure all quantities in units of the Planck units (and multiples thereof), t_p, $l_p = ct_p$, $m_p = \hbar/(cl_p)$, $T_p = m_p c^2/k_B$, and $I_p = (\varepsilon_0 \hbar c)^{1/2}/t_p$. Under these conventions, any time-variation of the fine-structure constant implies the variation of the unit electric charge e.

[1] P. A. M. Dirac, *Nature* **139**, 323 (1937).
[2] P. A. M. Dirac, *Proc. R. Soc. A* **165**, 199 (1938).
[3] E. A. Milne, *Proc. R. Soc. A* **158**, 324 (1937).
[4] P. Jordan, *Die Herkunft der Sterne* (Stuttgart, Vieweg, 1947).
[5] E. Teller, *Phys. Rev.* **73**, 801 (1948).
[6] R. H. Dicke, *Nature* **192**, 440 (1961).
[7] F. J. Dyson, *Aspects of Quantum Theory*, eds. A. Salam and E. P. Wigner (Cambridge, Univ. Press, 1972), p. 213.
[8] B. Carter, *Confrontation of Cosmological Theories with Observation*, ed. M. S. Longair (Dordrecht, Reidel, 1974), p. 291.
[9] B. J. Carr and M. J. Rees, *Nature* **278**, 605 (1979).
[10] A. V. Ivanchik, A. Y. Potekhin, and D. A. Varshalovich, *Astron. Astrophys.* **343**, 439 (1999).
[11] B. S. Deaver and W. M. Fairbank, *Phys. Rev. Lett.* **7**, 43 (1961).
[12] R. Doll and M. Näbauer, *Phys. Rev. Lett.* **7**, 51 (1961).
[13] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, *Phys. Rev.* **108**, 1175 (1957).
[14] B. D. Josephson, *Phys. Lett.* **1**, 251 (1962).
[15] Y. Aharonov and D. Bohm, *Phys. Rev.* **115**, 485 (1959).
[16] R. G. Chambers, *Phys. Rev. Lett.* **5**, 3 (1960).
[17] K. von Klitzing, G. Dorda, and M. Pepper, *Phys. Rev. Lett.* **45**, 494 (1980).
[18] D. C. Tsui, H. L. Störmer, and A. C. Gossard, *Phys. Rev. Lett.* **48**, 1559 (1982).
[19] S. Chandrasekhar, *Astrophys. J.* **74**, 81 (1931).
[20] M. Planck, *S.-B. Preuss. Akad. Wiss.* (1899), p. 440.
[21] A. Einstein, *Ann. Phys. (Leipzig)* **49**, 769 (1916).
[22] E. P. Hubble, *Proc. Natl. Acad. Sci.* **15**, 168 (1929).
[23] J. K. Webb et al., *Phys. Rev. Lett.* **82**, 884 (1999).
[24] W. de Sitter, *Konin. Ned. Akad. Wetenschappen* **19**, 1217 (1917).
[25] H. Bondi and T. Gold, *Mon. Not. R. Astron. Soc.* **108**, 252 (1948).
[26] F. Hoyle, *Mon. Not. R. Astron. Soc.* **108**, 372 (1948).
[27] P. J. E. Peebles et al., *Nature* **352**, 769 (1991).
[28] A. Friedmann, *Z. Phys.* **10**, 377 (1922).
[29] A. Friedmann, *Z. Phys.* **21**, 326 (1924).
[30] G. Lemaître, *Ann. Soc. Sci. Brux.* **47**, 49 (1927).
[31] J. Hoell, D.-E. Liebscher, and W. Priester, *Astron. Nachr.* **315**, 89 (1994).
[32] A. Einstein and W. de Sitter, *Proc. Natl. Acad. Sci* **18**, 213 (1932).
[33] M. Persic and P. Salucci, *Mon. Not. R. Astron. Soc.* **258**, 14 (1992).
[34] A. Vilenkin, *Phys. Rept.* **121**, 263 (1985).
[35] N. Hata et al., *Phys. Rev. Lett.* **75**, 3977 (1995).
[36] C. J. Copi, D. N. Schramm, and M. S. Turner, *Phys. Rev. Lett.* **75**, 3981 (1995).
[37] M. J. Pierce et al., *Nature* **371**, 385 (1994).
[38] W. L. Freedman et al., *Nature* **371**, 757 (1994).
[39] N. R. Tanvir et al., *Nature* **377**, 27 (1995).
[40] W. E. Harris et al., *Nature* **395**, 45 (1998).
[41] B. F. Madore et al., *Nature* **395**, 47 (1998).
[42] D.-E. Liebscher, W. Priester, and J. Hoell, *Astron. Astrophys.* **261**, 377 (1992).
[43] D.-E. Liebscher, W. Priester, and J. Hoell, *Astron. Nachr.* **313**, 265 (1992).
[44] C. van de Bruck, *Phys. Rev. D* **57**, 1306 (1998).
[45] C. van de Bruck, *Phys. Rev. D* **57**, 4663 (1998).
[46] D. V. Ahluwalia, unpublished (1999).